

The Vacuum Energy from a New Perspective

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Abstract. It is commonly believed that the vacuum energy problem points to the need for (1) a radically new formulation of gravitational physics and (2) a new principle which forces the vacuum stress-energy tensor (as measured by gravity) to be nearly zero. Here we point out that a new fundamental theory contains both features: (1) In this theory the vierbein is interpreted as the “superfluid velocity” associated with the order parameter Ψ_s for a GUT-scale Higgs condensate. (2) The vacuum stress-energy tensor $\mathcal{T}_{\mu\nu}^{vac}$ is exactly zero in the vacuum state, because the action is extremalized with respect to variations in Ψ_s . With inhomogeneously-distributed matter present, $\mathcal{T}_{\mu\nu}^{vac}$ is shifted away from zero.

The vacuum energy is one of the deepest issues in theoretical physics, and no conventional theory – including superstring/M theory – has offered a convincing solution to the problem of why the vacuum stress-energy tensor (as measured by gravity) is vastly smaller than expected but still nonzero [1-3].

In this paper we consider an unconventional theory which contains a radically new formulation of gravitational physics [4-6]. The gravitational vierbein is interpreted as the “superfluid velocity” of a GUT-scale condensate Ψ_s which forms in the very early universe:

$$g^{\mu\nu} = \eta^{\alpha\beta} e_\alpha^\mu e_\beta^\nu \quad , \quad e_\alpha^\mu = v_\alpha^\mu \quad , \quad v^\mu = v_\alpha^\mu \sigma^\alpha \quad \text{with} \quad \mu, \alpha = 0, 1, 2, 3 \quad (1)$$

$$v^\mu = \eta^{\mu\nu} v_\nu \quad , \quad m v_\mu = -i U^{-1} \partial_\mu U \quad , \quad \Psi_s = n_s^{1/2} U \eta_s \quad , \quad \eta_s^\dagger \eta_s = 1. \quad (2)$$

Here $\eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ is the Minkowski metric tensor, the σ^α are the identity matrix and three Pauli matrices, U is a 2×2 unitary matrix, η_s is a constant 2-component vector, and n_s is the condensate density. (After a Kaluza-Klein reduction from a higher-dimensional theory, the initial group of this order parameter is $SO(10) \times SU(2) \times U(1)$. For the purposes of this paper, however, the gauge group $SO(10)$ can be ignored, leaving the simpler description of (1) and (2).) In the present theory, Ψ_s is not static but instead exhibits $SU(2) \times U(1)$ rotations as a function of position and time. This condensate also supports Planck-scale $SU(2)$ instantons (in a Euclidean picture), which are analogous to the $U(1)$ vortices in an ordinary superfluid. In the present theory, the Einstein-Hilbert action and the curvature of spacetime result from these instantons [4]. Quantum gravity has a natural cutoff at the energy scale $m \sim$ the Planck energy m_P , but at lower energies one recovers the Einstein field equations

$$\frac{\delta S_{total}}{\delta g^{\mu\nu}} = \frac{\delta S_{vac}}{\delta g^{\mu\nu}} + \frac{\delta S_{fields}}{\delta g^{\mu\nu}} + \frac{\delta S_{EH}}{\delta g^{\mu\nu}} = 0 \quad (3)$$

or

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}^{(4)}R = \frac{1}{2}\ell_P^2\mathcal{T}_{\mu\nu}^{total} = \frac{1}{2}\ell_P^2\left(\mathcal{T}_{\mu\nu}^{vac} + \mathcal{T}_{\mu\nu}^{fields}\right) \quad (4)$$

where

$$\mathcal{T}_{\mu\nu}^{vac} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{vac}}{\delta g^{\mu\nu}} \quad , \quad \mathcal{T}_{\mu\nu}^{fields} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{fields}}{\delta g^{\mu\nu}} \quad (5)$$

and ℓ_P is the Planck length defined in Ref. 4.

In the vacuum state, there is no stress-energy tensor for matter and radiation: $\mathcal{T}_{\mu\nu}^{fields} = 0$. We additionally assume that there is no contribution from topological defects in the vacuum state: $\delta S_{EH}/\delta g^{\mu\nu} = 0$. This assumption will be discussed in more detail elsewhere [6], but it will be seen below that it leads to a consistent solution (whereas such a solution could not be obtained in conventional physics). We then have

$$\delta S_{vac} = \delta S_{total} = 0 \quad \text{in the vacuum state} \quad (6)$$

for arbitrary variations of the order parameter Ψ_s . Variations in v_α^μ , however, are a special case of functional variations in Ψ_s . It follows that the vacuum stress-energy tensor is exactly zero:

$$\mathcal{T}_{\mu\nu}^{vac} = 0 \quad \text{in the vacuum state.} \quad (7)$$

It is interesting to see in more detail how (7) can be achieved. According to the quantum Bernoulli equation (3.20) of Ref. 4, we have

$$\frac{1}{2}m\eta_s^\dagger\eta_{\mu\nu}v^\mu v^\nu\eta_s + V + P + V_{vac} = \mu \quad (8)$$

$$V = bn_s \quad , \quad P = -\frac{1}{2m}n_s^{-1/2}\eta^{\mu\nu}\partial_\mu\partial_\nu n_s^{1/2} \quad (9)$$

where μ is a fundamental energy which plays the role of a chemical potential here and which is comparable to m_P . We have added a term V_{vac} which represents the contribution of all other vacuum fields to δS_{vac} when Ψ_s is varied. Let us rewrite (8) as

$$-\frac{1}{2m}n_s^{-1/2}\eta^{\mu\nu}\partial_\mu\partial_\nu n_s^{1/2} + bn_s = \mu - V_{vac} - \frac{1}{2}m\eta_{\mu\nu}e_\alpha^\mu e_\beta^\nu\eta_s^\dagger\sigma^\alpha\sigma^\beta\eta_s. \quad (10)$$

After V_{vac} and e_α^μ are specified, the condensate density n_s adjusts itself to satisfy (10), (6), and (7). One might express this result as follows: In the vacuum state, the vacuum stress-energy tensor is tuned to exactly zero through adjustments of the condensate density. Notice that the extremalization (3) in conventional physics requires a contribution from the Einstein-Hilbert action S_{EH} even when $\delta S_{fields}/\delta g^{\mu\nu} = 0$, but in the present theory this extremalization in the vacuum state can be accomplished with S_{vac} alone.

In a more general state with matter, radiation, and topological defects present, it is the *total* action which is extremalized in (3). Then $\delta S_{vac}/\delta g^{\mu\nu}$ is shifted away from zero:

$$\mathcal{T}_{\mu\nu}^{vac} \neq 0 \quad \text{with matter and radiation present.} \quad (11)$$

There are two primary aspects of the vacuum energy problem [1, 2]: (i) Why is the vacuum stress-energy tensor many orders of magnitude smaller than predicted by conventional physics? This question is addressed in (7). (ii) Why is the vacuum stress-energy tensor not exactly zero? This is addressed in (11).

Although these are the “big” questions, one can add two more in the present context: (iii) Why was the vacuum energy density small compared to the density of matter and radiation during the period of big-bang nucleosynthesis? (iv) Why is it comparable to the density of matter now? To fully answer these questions will require a detailed treatment of how the vacuum energy is affected by the presence of matter and radiation. Suppose, however, that the dominant mechanism is a Casimir-like effect, in which the vacuum energy is modified by the boundary conditions imposed on the vacuum fields when there is an inhomogeneous distribution of matter. In a radiation-dominated universe, the energy density will be relatively homogeneous, and such an effect should be relatively small. In the present epoch, on the other hand, there is an extremely inhomogeneous distribution of matter. This plausibility argument indicates that the vacuum energy should play an important role only in the present epoch, and that the vacuum energy density (as measured by the stress-energy tensor) will be comparable to the density of inhomogeneously-distributed matter.

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References

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